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EFFECT OF VISCOSITY AND PRESTRESSED ONLOVE WAVES PROPAGATION

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ABSTRACT

The effect of initial stress and viscosity on the propagating love waves in a prestressed viscoelastic layer over a rigid half space is considered. It is shown that wave number group velocity and phase velocity are highly affected by initial stresses and viscosity.

The Paper presents a study on Love wave's propagation in a prestressed viscoelastic layer over laying a right half-space. The characteristic frequency equation is obtained and the variation of wave number with frequency under the combined effect of initial stress and viscoelasticity is analysed in detail.

INTRODUCTION

In the early 1900s the discovery of surface waves in a layer over a half-space for homogeneous isotropic medium by Love, about 100 years ago, many researchers have been studied extensively this problem for constant or variable velocity, rigidity and density. Most of the work has summarized in the book of Ewing et al (1957).

Waves propagating through elastic medium carry lot of information with them which are useful in several fields engineering, seismology, environmental sciences, matter sciences etc. The signal when pass through the medium provides us minor /major characteristics, i.e. discontinuity present in it homogeneity or in homogeneity, isotropic or anisotropy etc.

The study of Love waves is important to seismologist for its possible application in prediction of earth structure. The propagation behaviour of surface waves in a viscoelastic and anisotropic models is of great interest for the accurate inversion of the observed surface wave's data.

Schwab and Knopoff (1972) and Cramping and Taylor (1971) have shown that viscoelasticity and anisotropy have a considerable Influence on the propagation of surface waves. Gir Subhash and Gaur (1978) studied the propagation of Love waves in an anisotropic viscoelastic layer overlying a rigid half-space. The variation of the wave number with frequency under the combined effect of visco-elasticity and anisotropy is analysed in detail. Since the earth is an initially stressed body, it should be interesting to see how the velocity of Love waves is influenced by inherent initial stresses. Dey and Addy (1978) showed the effect of initial stresses on the frequency equation of Love waves in a visco-elastic medium overlying elastic half-space. They proved that the effect of viscosity is neglected, the phase velocity of Love waves decreases with increasing horizontal hydrostatic stresses.

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BASIC EQUATIONS

When body forces are not considered, the dynamical equations of equilibrium with normal initial stress $P = -S_{11}$ along the horizontal direction are given by Biot (1965) :

$$\frac{\partial s_{11}}{\partial x} + \frac{\partial s_{12}}{\partial y} + \frac{\partial s_{13}}{\partial z} - P\frac{\partial \omega_z}{\partial y} + P\frac{\partial \omega_y}{\partial z} = \rho\frac{\partial^2 u}{\partial t^2} \ ,$$

$$\frac{\partial s_{21}}{\partial x} + \frac{\partial s_{22}}{\partial y} + \frac{\partial s_{23}}{\partial z} - \rho\frac{\partial \omega_z}{\partial x} = \rho\frac{\partial^2 v}{\partial t^2} \ , \qquad (1)$$

$$\frac{\partial s_{31}}{\partial x} + \frac{\partial s_{32}}{\partial y} + \frac{\partial s_{33}}{\partial z} + P\frac{\partial \omega_y}{\partial x} = \rho\frac{\partial^2 \omega}{\partial t^2} \ ,$$

where $s_{ij}\ (i, j = 1,2,3)$ are the incremental stress components and $\omega_x, \omega_y, \omega_z$ are the rotational components given by

$$\omega_x = \frac{1}{2}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right),$$

$$\omega_y = \frac{1}{2}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right), \qquad (2)$$

$$\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right).$$

u, v *and* w are the displacement components along axes. ρ is the density of the medium. Stress – strain relation for an initially stressed orthotropic elastic medium with co-ordinate planes of elastic symmetry are .

$$s_{11} = B_{11}\frac{\partial u}{\partial x} + B_{12}\left(\frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right),$$

$$s_{22} = (B_{12} - P)\frac{\partial u}{\partial x} + B_{22}\frac{\partial v}{\partial y} + B_{23}\frac{\partial w}{\partial z} \ ,$$

$$s_{33} = (B_{12} - P)\frac{\partial u}{\partial x} + B_{23}\frac{\partial v}{\partial y} + B_{22}\frac{\partial w}{\partial z} \ , \qquad (3)$$

$$s_{23} = Q_1\left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial x}\right),$$

$$s_{31} = Q_2\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right),$$

$$s_{12} = Q_3\left(\frac{\partial u}{\partial y} + \frac{\partial u}{\partial x}\right).$$

$B_{ij}\ (i,j = 1,2,3); Q_i\ (i,j = 1,2,3)$ are the incremental elastic coefficients and shear moduli respectively. Where the elastic coefficients B_{ij}, Q_i of the prestressed solid are related to the Lam$\acute{e}$ constants λ, μ of the isotropic unstressed state and the prestrains ϵ_{ij} by means of explicit relations for either finite or infinitesimal deformations.In the linear approximation [Biot(1965)].

$$B_{11} = (2\mu + \lambda)(1 + \epsilon_{11} - 2\epsilon_{22}),$$

$$B_{22} = (2\mu + \lambda)(1 + \epsilon_{11}),$$

$$B_{12} = \lambda(1 - \epsilon_{22}) - S_{11},$$

$$B_{23} = \lambda(1 - \epsilon_{11}),$$

$$Q_1 = \mu + (\mu + \lambda)\epsilon_{22} + \frac{1}{2}(\lambda - 2\mu)\,\epsilon_{11},$$

$$Q_3 = Q_2 = \mu + \frac{1}{2}(\mu + \lambda)(\epsilon_{11} + \epsilon_{22}) + \frac{1}{2}(\mu - 2\mu)\epsilon_{22}\,.$$

The equilibrium equations in terms of displacement components can be obtained from eqnations (1) –(3),we get

$$B_{11}\frac{\partial^2 u}{\partial x^2} + \left(Q_3 + \frac{P}{2}\right)\frac{\partial^2 u}{\partial y^2} + \left(Q_2 + \frac{P}{2}\right)\frac{\partial^2 u}{\partial z^2} + \left(B_{12} + Q_3 - \frac{P}{2}\right)\frac{\partial^2 u}{\partial x \partial y} + \left(B_{12} + Q_3 - \frac{P}{2}\right)\frac{\partial^2 w}{\partial x \partial z} = \rho\frac{\partial^2 u}{\partial t^2},$$

$$\left(B_{12} + Q_3 - \frac{P}{2}\right)\frac{\partial^2 u}{\partial x \partial y} + \left(Q_3 - \frac{P}{2}\right)\frac{\partial^2 v}{\partial x^2} + B_{22}\frac{\partial^2 v}{\partial y^2} + Q_1\frac{\partial^2 v}{\partial z^2} + (B_{23} + Q_1)\frac{\partial^2 w}{\partial y \partial z} = \rho\frac{\partial^2 v}{\partial t^2}, \qquad (4)$$

$$\left(B_{12} + Q_3 - \frac{P}{2}\right)\frac{\partial^2 u}{\partial x \partial z} + (Q_1 + B_{23})\frac{\partial^2 v}{\partial y \partial z} + B_{22}\frac{\partial^2 w}{\partial z^2} + \left(Q_2 - \frac{P}{2}\right)\frac{\partial^2 w}{\partial x^2} + Q_1\frac{\partial^2 w}{\partial y^2} = \rho\frac{\partial^2 u}{\partial t^2}\,.$$

FORMULATION OF PROBLEM

Let us consider a prestressed voigt-type viscoelastic layer of thickness H overlying a rigid half-space with constant normal initial stress along the horizontal direction.

we take rectangular co-ordinate system with z-axis directed vertically downward .The surface of half-space is located at $z = 0$ and $x - axis$ is taken horizontally as shown in Figure1.

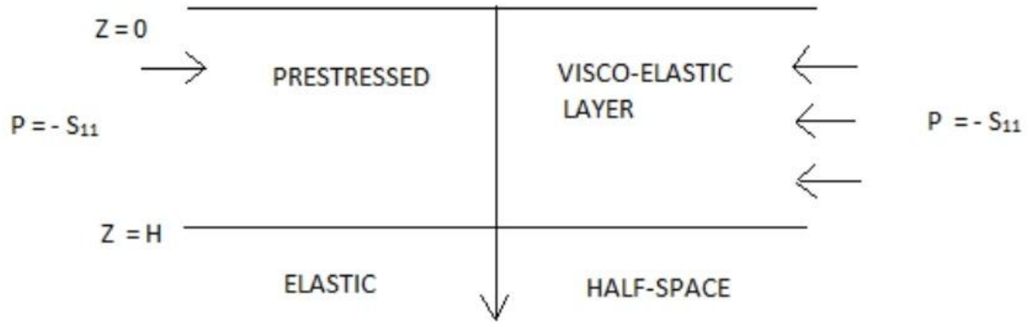

Figure.1

We consider Love waves propagation in the positive x –direction .The displacement components are independent of y and are of the type

$$u = 0, v = v(x,z), w = 0 \; and \quad \frac{\partial}{\partial y} \equiv 0. \quad (5)$$

Putting equation(5) in equation(4),we get

$$\left(Q_3 - \frac{P}{2}\right)\frac{\partial^2 v}{\partial t^2} + Q_1 \frac{\partial^2 v}{\partial z^2} = \rho \frac{\partial^2 v}{\partial t^2}, \qquad (6)$$

But here layer is prestressed visco-elastic medium so the equation (6) of motion for Love waves becomes

$$(Q_3 - \frac{P}{2} + Q_3' \frac{\partial}{\partial t})\frac{\partial^2 v}{\partial x^2} + (Q_1 - \frac{P}{2} + Q_1' \frac{\partial}{\partial t})\frac{\partial^2 v}{\partial z^2} = \rho \frac{\partial^2 v}{\partial t^2} \qquad (7)$$

Where $Q_3 \; and \; Q_3'$ are viscoelastic parameters.For unstressed case $Q_3 = Q_1 = \mu$.

For a plane harmonic wave propagating in the x –direction we put

$$v = V(z)\exp\left[i(\omega t - kx)\right], \qquad (8)$$

where $V(z)$ is a real function of $z \; and \; k$ is a complex quantity equal to $(K_0 + iK_1)$,whose real part represents the wave number and the imaginary part represents the damping coefficient.ω is a fixed positive circular frequency.Putting this value of $V(z)$ in equation(7),we get

$$(Q_1 + Q_1'\omega i)\frac{\partial^2 V}{\partial z^2} + [\rho\omega^2 - K^2\overline{(Q_3 - \frac{P}{2} + Q_3'\omega i)}]V = 0 \quad . \; (9)$$

The above equation reduces to

$$\frac{\partial^2 V}{\partial z^2} + S^2 V = 0 \qquad (10)$$

and its solution can be written as

$$\bar{V} = A\,exp(iSz) + B\,exp(-iSz). \qquad (11)$$

Where A and B are constants and S is complex quantity given by

$$S^2 = \frac{\left[\rho\omega^2 - K^2\left(Q_3 - \frac{P}{2} + Q_3^{'}\omega i\right)\right]}{\left(Q_1 + Q_1^{'}\omega i\right)} . \qquad (12)$$

Equations(8) and (12) give

$$v = [Aexp(iSz) + Bexp(-iSz)]\,\exp[i(\omega t - kx)].$$

BOUNDARY CONDITIONS

Stress at the free surface and displacement at the interface are zero.

$$\frac{\partial v}{\partial z} = 0 \quad at \quad z = 0 \qquad (13)$$

$$\text{and } V = 0 \quad at \ Z = H \qquad (14)$$

From equations (11),(13) and (16) we get

$$A = B \qquad (15)$$

$$A\,exp(iSH) + B\,exp(-iSH) = 0 \qquad (16)$$

equations (15) and (16) on simplification, give

$$S^2H^2 = \left(n + \frac{1}{2}\right)^2 \pi^2 \qquad (17)$$

Putting the value of S^2 from equation(12) in equation(17) and separating into real and imaginary part, we get

$$RH^2\left[\omega^2\left(\frac{1}{\beta^2} - apQ^2\right) - a - b\omega Q(p-1) + a\frac{P}{2Q_3} + \frac{bPQW}{2Q_3}\right]$$

$$-iRH^2\left[\omega^2\left(\frac{Q\omega}{\beta^2} + pbQ^2\right) - b\frac{P}{2Q_3} + \frac{aPQW}{2Q_3} + b + a\omega Q(p-1)\right] = C\left(n + \frac{1}{2}\right)^2 \pi^2 \qquad (18)$$

where $\frac{Q_1^{'}}{Q_1} = Q$ and $\frac{Q_3^{'}}{Q_3} = pQ$. $\qquad (19)$

$\frac{1}{Q}$ represents the specific dissipation factor for shear wave.

$$R = \frac{Q_3}{Q_1}, \beta = \sqrt{\frac{Q_3}{\rho}}\,, C = (1 + Q^2\omega^2), \qquad (20)$$

and

$$a = K_0^2 - K_1^2\ , \qquad b = 2K_0K_1\,. \qquad (21)$$

Equating real and imaginary parts and putting $p = 1$,in equation (18), we get

$$RH^2\left[\omega^2\left(\frac{1}{\beta^2} - apQ^2\right) - a + a\frac{P}{2Q_3} + \frac{bPQW}{2Q_3}\right] = C\left(n + \frac{1}{2}\right)^2 \pi^2, (22)$$

$$\left[\omega^2\left(\frac{Q\omega}{\beta^2} + bQ^2\right) - b\frac{P}{2Q_3} - \frac{aPQW}{2Q_3} + b\right] = 0 \quad . (23)$$

The values of $K_0(wave\ number)$ and $K_1(damping\ Coefficient)$ for various models are by the relations

$$K_0(n) = \sqrt{\tfrac{1}{2}\{(\sqrt{a^2+b^2}) + a\}} \qquad (24)$$

and

$$K_1(n) = \sqrt{\tfrac{1}{2}\{(\sqrt{a^2+b^2}) - a\}} \qquad (25)$$

From Equations(22),It is clear that the propagation of Love waves should take place at all frequencies without cut.

When $Q \ll 1$,the contribution of b at low frequencies will be near to zero and therefor the dispersion behaviour of Love waves will be same as for elastic earth.

PARTICULAR CASE

Let us consider a Voigt-type viscoelastic medium of thickness H over an elastic-half-space. Put $P = -S_{11}$= 0 in the equations(22) and (23) we get

$$RH^2\left[\left(\left(\frac{\omega}{\beta}\right)\right)^2 - aC\right] = \left(n + \frac{1}{2}\right)^2 \pi^2 C\ , \qquad (26)$$

$$\frac{\omega^3 Q}{\beta^2} + bC = 0\ . \qquad (27)$$

These equations coincide with the equations(16) and (17) of Subhash and Gaur.

Equation (21) along with equations (19) and (20) has been used to analyse the wave number for following two cases assuming $Q = 50$.

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ISOTROPIC VISCOELASTIC CASES (PUT $P = 0$, R=1)

It is clear from Table 1. That for a viscoelastic surface layer, the value of KH is smaller than its corresponding value for a perfectly elastic layer. The effect of viscoelasticity is therefore todecrease the wave number.

ANISOTROPIC VISCOELASTIC CASE (PG 0, O G 0)

Comparing the results given in the figure 2 and figure 3, it can be observed that where as the effect of initial stresses is to increase the wave number value and viscoelastic tends to decrease it. Two effects therefore counteract each other. Since the effect of viscoelasticity goes on increasing with frequency.

		.5	**.8**	**1.0**	**2.0**	**3.0**	**5.0**	**8.0**
n=0	KH(Q=0) KH(Q=50)	0.6701 0.2726	1.0169 0.2743	**1.2546** 0.2767	**2.4646** **0.3136**	**3.6843** 0.3830	**6.1298** 0.5577	**9.8017** 0.8460
n=1		1.0204 0.8163	1.2752 0.8165	1.4718 0.8167	2.5819 0.8199	3.7638 0.8301	6.1779 0.8898	9.831 1.0729
n=2		1.4918 1.3604	1.6765 1.3605	1.8305 1.3603	2.8019 1.3617	3.9180 1.3647	6.2730 1.3864	9.8919 1.4686
n=3		2.0005 1.9045	2.1417 1.9046	2.2643 1.9047	3.1028 1.9053	4.1385 1.9068	6.4130 1.9151	9.9813 1.9512

Table 1.

Figure1.

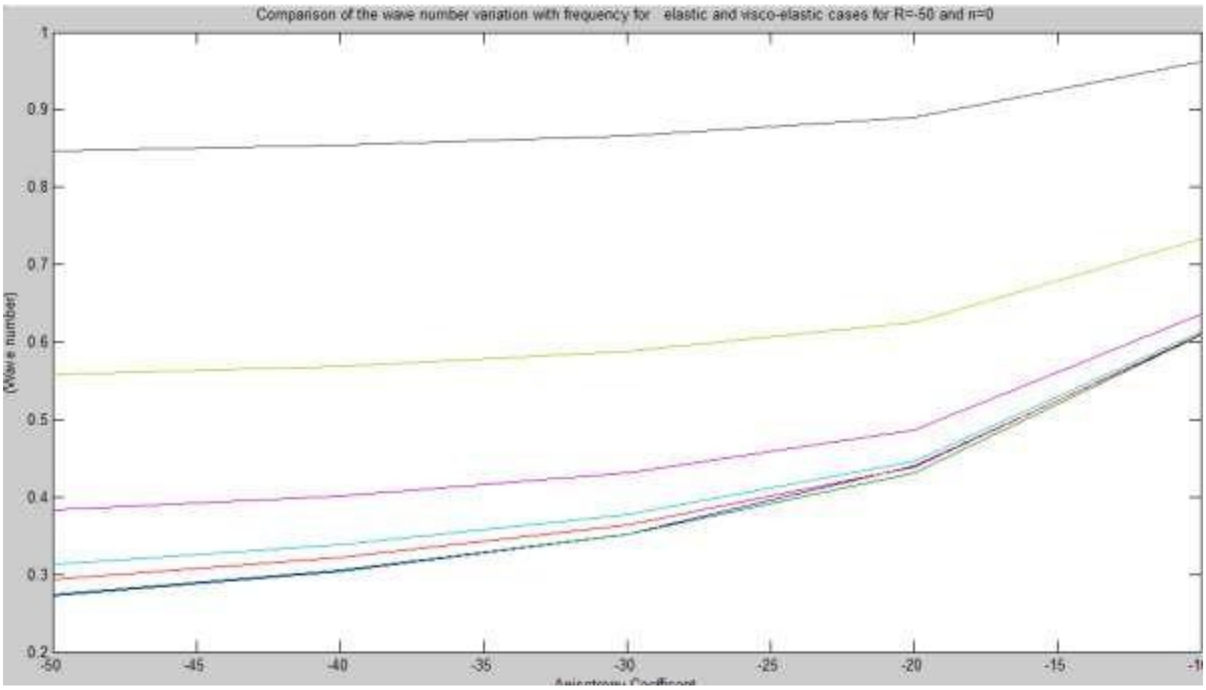

Figure 2

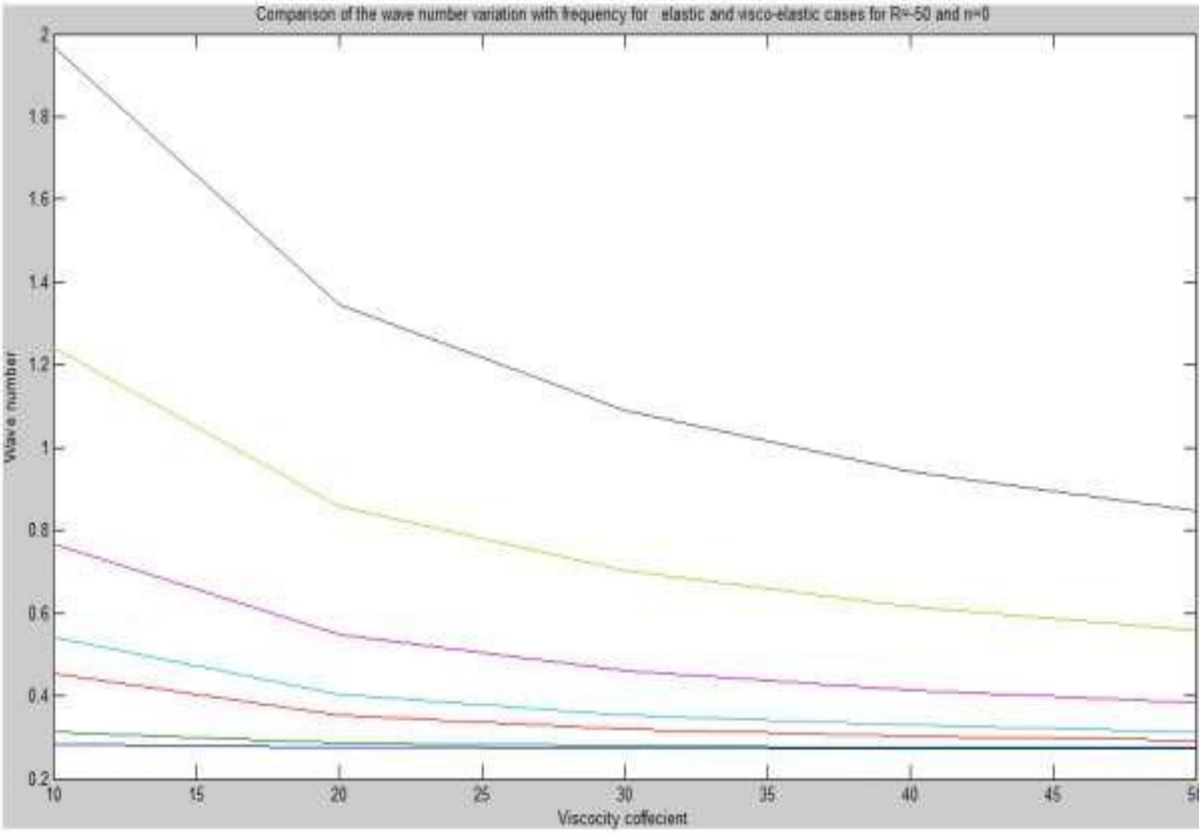

Figure 3

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